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THE TRIPLE LAPLACE TRANSFORMS AND THEIR PROPERTIES

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ABSTRACT

This paper deals with the triple Laplace transforms and their properties with examples and applications to functional, integral and partial differential equations. Several simple theorems dealing with general properties of the triple Laplace transform are proved. The convolution, its properties and convolution theorem with a proof are discussed in some detail. The main focus of this paper is to develop the method of the triple Laplace transform to solve initial and boundary value problems in applied mathematics, and mathematical physics.

KEYWORDS: *Triple Laplace Transform, Double Laplace Transform, Single Laplace Transform, Convolution*

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1.1. INTRODUCTION

P. S. Laplace (1749–1827) introduced the idea of the Laplace transform in 1782, In his celebrated study of probability theory and celestial mechanics. Laplace's classic treatise on LaTh´eorie Analytique des Probabilities (Analytical Theory of Probability) contained some basic results of the Laplace transform which is one of the oldest and most commonly used linear integral transforms available in the mathematical literature. This has effectively been used in finding the solutions of linear differential, difference and integral equations. On the other hand, Joseph Fourier's (1768–1830) monumental treatise on La Th´eorieAnalytique de la Chaleur (The Analytical Theory of Heat) provided the modern mathematical theory of heat conduction, Fourier series, and Fourier integrals with applications. In his treatise, he discovered a double integral representation of a non-periodic function f(x) for all real x which is universally known as the Fourier Integral Theorem in the form

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}\left[\int_{-\infty}^{\infty} f(\xi)e^{-ik\xi}d\xi\right]dk$$

The deep significance of this theorem has been recognized by mathematicians and mathematical physicists of the nineteenth and twentieth centuries. Indeed, this theorem is regarded as one of the most fundamental representation theorems of modern mathematical analysis and has widespread mathematical, physical and engineering applications. According to Lord Kelvin (1824–1907) and Peter Guthrie Tait (1831–1901) once said: "Fourier's Theorem, which is not only one of the most beautiful results of modern analysis, but may be said to furnish an indispensable instrument in the treatment of nearly recondite question in modern physics". Another remarkable fact is that the Fourier integral theorem was used by Fourier to introduce the Fourier transform and the inverse Fourier transform. This celebrated work of Fourier was known to Laplace, and, in fact, the Laplace transform is a special case of the Fourier transform. It was also A. L. Cauchy (1789–1857) who also used independently some of the ideas of the theory of Fourier transforms. At the same time, S. D. Poisson (1781–1840) also independently applied the method of Fourier transforms in his research on the propagation of water waves. Although both Laplace and Fourier transforms have been discovered in the 19th century, it was the British electrical engineer, Oliver Heaviside (1850–1925) who made the Laplace transform very popular by applying it to solve ordinary differential equations of electrical circuits and systems, and then to develop modern operational calculus in less rigorous way. He first recognized the power and success of his operational method and then used it as a powerful and effective tool for the solutions of telegraph equations and the second order hyperbolic partial differential equations with constant coefficients. Subsequently, T. J. Bromwich (1875–1930) first successfully introduced the theory of complex functions to provide formal mathematical justification of Heaviside's

Operational calculus. After Bromwich's work, notable contributions to rigorous formulation of operational calculus were made by J. R. Carson (1886–1940), Van der Pol, (1892–1977), G. Doetsch (1889–1959) and many others. Both Laplace and Fourier transforms have been studied very extensively and have found to have a wide variety of applications in mathematical, physical, statistical, and engineering

Sciences and also in other sciences. At present, there is very little work available on the double Laplace transforms of $f(x, y)$ of two positive real variables x and y and their properties and applications. So, the major objective of this paper is to study the triple Laplace transform, its properties with examples and applications to functional, integral and partial differential equations. Several simple theorems dealing with general properties of the double Laplace theorem areproved. The convolution of $f(x, y, z)$ and $g(x, y, z)$, its properties and convolution theorem with a proof are discussed in some detail.Thakur(2015) have develop Some Properties of Triple Laplace Transform. The main focus of this paper is to develop the method of the triple Laplace transform to solve initial and boundary value problems in applied mathematics, and mathematical physics.

1.2. Definition of the Triple Laplace Transform and Examples

The triple Laplace transform of a function $F(x, y, z)$ of three variables x, y and z defined in the first octane of the xyz - plane is defined by the triple integral in the form

$$f(p,q,r) = L_3\left[F(x,y,z)\right] = L\left[L\left\{L\left(F(x,y,z);x \to p\right);y \to q\right\};z \to r\right]$$

$$\therefore\ f(p,q,r) = L_3\left[F(x,y,z)\right] = \int_0^\infty\int_0^\infty\int_0^\infty F(x, y, z)e^{-(px+qy+rz)}dxdydz$$

Provided the integral exists, where we follow Debnath and Bhatta to denote the Laplace transform $f(p) = L\{\mathrm{F}(x);x \to p\}$ of $f(x)$ and to define by

$$f(p) = L\{\mathrm{F}(x)\} = \int_0^\infty e^{-px}F(x)dx\ ,\ \mathrm{Re}(p) > 0$$

and the inverse Laplace transformation of $f(p)$ and to define by

$$F(x) = L^{-1}\{f(p)\} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{px} f(p)\,dp \quad , c \geq 0$$

Evidently, L_3 is a linear integral transformation as shown below :

$$L_3\,[a_1F_1(x, y, z) + a_2F_2(x, y, z)] =$$
$$\int_0^\infty\int_0^\infty\int_0^\infty [a_1F_1(x, y, z) + a_2F_2(x, y, z)]\,e^{-(px+qy+rz)}dxdydz$$

$$= \int_0^\infty\int_0^\infty\int_0^\infty a_1F_1(x, y, z)\,e^{-(px+qy+rz)}dxdydz + \int_0^\infty\int_0^\infty\int_0^\infty a_2F_2(x, y, z)\,e^{-(px+qy+rz)}dxdydz$$

$$= a_1\int_0^\infty\int_0^\infty\int_0^\infty F_1(x, y, z)\,e^{-(px+qy+rz)}dxdydz + a_2\int_0^\infty\int_0^\infty\int_0^\infty F_2(x, y, z)\,e^{-(px+qy+rz)}dxdydz$$

$$= a_1L_3\,[\mathrm{F}_1(x, y, z)] + a_2L_3[\mathrm{F}_2(x, y, z)]$$

Where a_1 and a_2 are constants.

The inverse triple Laplace transform $L_3^{-1}\left[f(p,q,r)\right] = F(x, y, z)$ is defined by the triple integral formula

$$L_3^{-1}\left[f(p,q,r)\right] = F(x, y, z)$$

$$= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{px}\,dp\,\frac{1}{2\pi i}\int_{d-i\infty}^{d+i\infty} e^{qy}\,dq\,\frac{1}{2\pi i}\int_{e-i\infty}^{e+i\infty} e^{rz}\,f(p,q,r)\,dr$$

It follows that $L_3^{-1}\left[f(p,q,r)\right]$ satisfies the linear property

$$L_3^{-1}\left[a\,f(p,q,r) + b\,g(p,q,r)\right] = a\,L_3^{-1}\left[\,f(p,q,r)\right] + b\,L_3^{-1}\left[\,g(p,q,r)\right].$$

where a and b are constants. This shows that L_3^{-1} is also a linear transformation.

Examples

(a) If $F(x, y, z) = k$ for $x > 0$, $y > 0$ and $z > 0$ then

$$f(p,q,r) = L_3\{k\} = L\left[L\{L(\mathrm{k}; x \to p); y \to q\}; z \to r\right]$$

$$f(p,q,r) = L_3\{k\} = k\int_0^\infty\int_0^\infty\int_0^\infty e^{-(px+qy+rz)}dxdydz$$

$$= k\int_0^\infty e^{-px}dx\int_0^\infty e^{-qy}dy\int_0^\infty e^{-rz}dz = \frac{k}{pqr}.$$

(b) If $F(x, y, z) = exp(ax+by+cz)$ for all x, y and z, then

$$f(p,q,r) = L_3\{exp(ax+by+cz)\} = \int_0^\infty\int_0^\infty\int_0^\infty e^{-\{(p-a)x+(q-b)y+(r-c)z)\}}dxdydz$$

$$= \int_0^\infty e^{-(p-a)x}dx\int_0^\infty e^{-(q-b)y}dy\int_0^\infty e^{-(r-c)z}dz$$

$$= \frac{1}{(p-a)(q-b)(r-c)}.$$

(c) Similarly If $F(x, y, z) = exp[i(ax+by+cz)]$, then

$$f(p,q,r) = L_3\{exp[i(ax+by+cz)]\} = \frac{1}{(p-ia)(q-ib)(r-ic)}$$

$$\Rightarrow L_3\{\cos(ax+by+cz)+\mathrm{i}\sin(ax+by+cz)\} = \frac{(p+ia)(q+ib)(r+ic)}{(p^2+a^2)(q^2+b^2)(r^2+c^2)}$$

$$\Rightarrow L_3\{\cos(ax+by+cz)\} + iL_3\{\sin(ax+by+cz)\}$$

$$= \frac{\{(pq-ab)r-(aq+bp)c\}+i\{(pq-ab)c+(aq+bp)r\}}{(p^2+a^2)(q^2+b^2)(r^2+c^2)}$$

Consequently,

$$L_3\{\cos(ax+by+cz)\} = \frac{(pq-ab)r-(aq+bp)c}{(p^2+a^2)(q^2+b^2)(r^2+c^2)}$$

And $$L_3\{\sin(ax+by+cz)\} = \frac{(pq-ab)c+(aq+bp)r}{(p^2+a^2)(q^2+b^2)(r^2+c^2)}.$$

If $a=b=c=1$, then

$$L_3\{\cos(x+y+z)\} = \frac{pqr-p-q-r}{(p^2+1)(q^2+1)(r^2+1)}$$

And $$L_3\{\sin(x+y+z)\} = \frac{pq+qr+rp-1}{(p^2+1)(q^2+1)(r^2+1)}.$$

(d) If $F(x,\ y,\ z) = cosh(ax+by+cz)$, then

$$L_3\{\cosh(ax+by+cz)\} = \frac{1}{2}\left[L_3\{exp(ax+by+cz)\}+L_3\{exp(-ax-by-cz)\}\right]$$

$$= \frac{1}{2}\left[\frac{1}{(p-a)(q-b)(r-c)}+\frac{1}{(p+a)(q+b)(r+c)}\right].$$

Similarly,

$$L_3\{\sinh(ax+by+cz)\} = \frac{1}{2}\left[L_3\{exp(ax+by+cz)\}-L_3\{exp(-ax-by-cz)\}\right]$$

$$= \frac{1}{2}\left[\frac{1}{(p-a)(q-b)(r-c)}-\frac{1}{(p+a)(q+b)(r+c)}\right].$$

(e) If $F(x,\ y,\ z) = (xyz)^n$, then

$$L_3\{(xyz)^n\} = \int_0^\infty\int_0^\infty\int_0^\infty (x\,y\,z)^n e^{-(px+qy+rz)}dxdydz$$

$$= \int_0^\infty x^n e^{-px}dx \int_0^\infty y^n e^{-qy}dy \int_0^\infty z^n e^{-rz}dz$$

$$= \frac{n!}{p^{n+1}}.\frac{n!}{q^{n+1}}.\frac{n!}{r^{n+1}} = \frac{(n!)^3}{p^{n+1}q^{n+1}r^{n+1}}.$$

where n is a positive integer.

Similarly,

$$L_3\{x^l\ y^m\ z^n\} = \frac{l!}{p^{l+1}}.\frac{m!}{q^{m+1}}.\frac{n!}{r^{n+1}} = \frac{l!m!n!}{p^{l+1}q^{m+1}r^{n+1}}.$$

We know that relation between Gama notation and factorial notation $\Gamma(s+1)=s!$

Then $L_3\left\{x^l\ y^m\ z^n\right\} = \dfrac{\Gamma(l+1)\Gamma(m+1)\Gamma(n+1)}{p^{l+1}q^{m+1}r^{n+1}}$.

where l, m and n are positive integers

Theorem 1: First Shifting Theorem: If $L_3\left[F(x,y,z)\right] = f(p,q,r)$, then $L_3\left[e^{-(ax+by+cz)}F(x,y,z)\right] = f\left((p+a),(q+b),(r+c)\right)$.

Proof: Let $L_3\left[F(x,y,z)\right] = f(p,q,r) = \int_0^\infty\int_0^\infty\int_0^\infty F(x,\ y,\ z)e^{-(px+qy+rz)}dxdydz$.

Then $L_3\left[e^{-(ax+by+cz)}F(x,y,z)\right] = \int_0^\infty\int_0^\infty\int_0^\infty F(x,\ y,\ z)e^{-(ax+by+cz)}\ e^{-(px+qy+rz)}dxdydz$

$$= \int_0^\infty\int_0^\infty\int_0^\infty e^{-\{(p+a)x+(q+b)y+(r+c)z)\}}\ F(x,y,z)dxdydz$$

$$= \int_0^\infty e^{-(p+a)x}\left[\int_0^\infty\int_0^\infty e^{-(q+b)y+(r+c)z}F(x,y,z)dydz\right]dx$$

Note that the integral inside the bracket satisfies the properties of the double

Laplace transform and is given as $\int_0^\infty\int_0^\infty e^{-(q+b)y+(r+c)z}F(x,y,z)dydz = f(x,q+b,r+c)$.

Thus $\int_0^\infty e^{-(p+a)x}F(x,q+b,r+c)dx = f(p+a,q+b,r+c)$.

Theorem 2: Change of Scale Property: If $L_3\left[F(x,y,z)\right] = f(p,q,r)$, then $L_3\left[F(ax,by,cz)\right] = \dfrac{1}{abc}f\left(\dfrac{p}{a},\dfrac{q}{b},\dfrac{r}{c}\right)$.

Proof: Let $L_3\left[F(x,y,z)\right] = f(p,q,r) = \int_0^\infty\int_0^\infty\int_0^\infty F(x,\ y,\ z)e^{-(px+qy+rz)}dxdydz$.

Then $L_3\left[F(ax, by, cz)\right] = \int_0^\infty\int_0^\infty\int_0^\infty F(ax,\ by, cz)e^{-(px+qy+rz)}dxdydz$

$$= \int_0^\infty e^{-px}\left[\int_0^\infty\int_0^\infty e^{-(qy+rz)}\ F(ax,\ by, cz)dydz\right]dx$$

Note that the integral inside the bracket satisfies the properties of the double Laplace transform and is given as

$$\int_0^\infty\int_0^\infty e^{-(qy+rz)}\ F(ax,\ by, cz)dydz = \frac{1}{bc} f\left(ax, \frac{q}{b}, \frac{r}{c}\right).$$

Thus $= \int_0^\infty e^{-px}\ \frac{1}{bc} F\left(ax, \frac{q}{b}, \frac{r}{c}\right)dx = \frac{1}{abc} f\left(\frac{p}{a}, \frac{q}{b}, \frac{r}{c}\right).$

Theorem 3: Multiplication by $x^l y^m z^n$: If $L_3\left[F(x,y,z)\right] = f(p,q,r)$, then

$$L_3\left[x^l y^m z^n F(x,y,z)\right] = (-1)^{l+m+n} \frac{\partial^{l+m+n} f(p,q,r)}{\partial p^l \partial q^m \partial r^n}.$$

Proof: Let $L_3\left[F(x,y,z)\right] = f(p,q,r) = \int_0^\infty\int_0^\infty\int_0^\infty F(x,\ y,\ z)e^{-(px+qy+rz)}dxdydz$.

Then $\frac{\partial^{l+m+n} f(p,q,r)}{\partial p^l \partial q^m \partial r^n} = \frac{\partial^{l+m+n}}{\partial p^l \partial q^m \partial r^n}\left(\int_0^\infty\int_0^\infty\int_0^\infty F(x,\ y,\ z)e^{-(px+qy+rz)}dxdydz\right)$

$$= \frac{\partial^l}{\partial p^l}\int_0^\infty e^{-px}\left(\frac{\partial^{m+n}}{\partial q^m \partial r^n}\int_0^\infty\int_0^\infty F(x,\ y,\ z)e^{-(qy+rz)}dydz\right)dx$$

Note that the integral inside the bracket satisfies the properties of the double

Laplace transform and is given by $\frac{\partial^{m+n}}{\partial q^m \partial r^n}\int_0^\infty\int_0^\infty F(x,\ y,\ z)e^{-(qy+rz)}dydz = L_2\left\{(-1)^{m+n}\ y^m z^n\ F(x,y,z)\right\}$

Thus $\frac{\partial^{l+m+n} f(p,q,r)}{\partial p^l \partial q^m \partial r^n} = \frac{\partial^l}{\partial p^l}\int_0^\infty e^{-px}\left(L_2\left\{(-1)^{m+n}\ y^m z^n\ F(x,y,z)\right\}\right)dx$

$$= L_3\left[(-1)^{l+m+n} x^l y^m z^n F(x,y,z)\right]$$

Therefore $L_3\left[x^l y^m z^n F(x,y,z)\right] = (-1)^{l+m+n} \dfrac{\partial^{l+m+n} f(p,q,r)}{\partial p^l \partial q^m \partial r^n}$.

Theorem 4: Assuming that the function $F(x, y, z)$ is triple Laplace transformable, then

(i) $L_3\left[\dfrac{\partial^3 F(x, y, z)}{\partial x \partial y \partial z}\right] = pqr\ f(p,q,r) - pq\ f(p,q,0) - pr\ f(p,0,r) - qr\ f(0,q,r)$

$+\ p\ f(p,0,0) + q\ f(0,q,0) + r\ f(0,0,r) - f(0,0,0)$.

(ii) $L_3\left[\dfrac{\partial^3 F(x, y, z)}{\partial x^3}\right] = p^3 f(p,y,z) - p^2 f(0,y,z) - p\dfrac{\partial f(0,y,z)}{\partial x} - \dfrac{\partial^2 f(0,y,z)}{\partial x^2}$.

Theorem 5: If $L_3\left[F(x,y,z)\right] = f(p,q,r)$, then

$L_3\left[F(x-\alpha, y-\beta, z-\gamma) H(x-\alpha, y-\beta, z-\gamma)\right] = e^{-(\alpha p+\beta q+\gamma r)} f(p,q,r)$, where

$H(x,y,z)$ is the Heaviside unit step function defined by $H(x-a, y-b, z-c) = 1$ when $x > a$,

$y > b$ and $z > c$: and $H(x-a, y-b, z-c) = 0$ when $x < a$, $y < b$ and $z < c$.

Proof: Let $L_3\left[F(x,y,z)\right] = f(p,q,r) = \int_0^\infty \int_0^\infty \int_0^\infty F(x, y, z) e^{-(px+qy+rz)} dxdydz$.

Then $L_3\left[F(x-\alpha, y-\beta, z-\gamma) H(x-\alpha, y-\beta, z-\gamma)\right]$

$$= \int_0^\infty \int_0^\infty \int_0^\infty e^{-(px+qy+rz)} F(x-\alpha, y-\beta, z-\gamma) H(x-\alpha, y-\beta, z-\gamma) dxdydz$$

$$= \int_\alpha^\infty \int_\beta^\infty \int_\gamma^\infty e^{-(px+qy+rz)} F(x-\alpha, y-\beta, z-\gamma) dxdydz$$

Which is, by putting $x-\alpha = u, y-\beta = v$ and $z-\gamma = w$

$$= e^{-(p\alpha+q\beta+r\gamma)} \int_0^\infty \int_0^\infty \int_0^\infty e^{-(pu+qv+rw)} F(u,v,w) dudvdw$$

$$= e^{-(p\alpha+q\beta+r\gamma)} f(p,q,r).$$

Theorem 6: If $F(x, y, z)$ is a periodic function of periods α, β and γ, i.e., $F(x+\alpha, y+\beta, z+\gamma) = F(x, y, z)$ and if $L_3[F(x,y,z)]$ exists, then $L_3[F(x,y,z)] = \frac{1}{1-e^{-(p\alpha+q\beta+r\gamma)}}\int_0^{\alpha}\int_0^{\beta}\int_0^{\gamma} e^{-(px+qy+rz)} F(x, y, z)dxdydz$.

Proof: Let $L_3[F(x,y,z)] = f(p,q,r) = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} F(x, y, z)e^{-(px+qy+rz)}dxdydz$

$$= \int_0^{\alpha}\int_0^{\beta}\int_0^{\gamma} e^{-(px+qy+rz)} F(x, y, z)dxdydz + \int_{\alpha}^{\infty}\int_{\beta}^{\infty}\int_{\gamma}^{\infty} e^{-(px+qy+rz)} F(x, y, z)dxdydz$$

Putting $x = u+\alpha, y = v+\beta$ and $z = w+\gamma$ in second triple integral, we obtain

$$f(p,q,r) = \int_0^{\alpha}\int_0^{\beta}\int_0^{\gamma} e^{-(px+qy+rz)} F(x, y, z)dxdydz$$

$$+ e^{-(p\alpha+q\beta+r\gamma)}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-(pu+qv+rw)} F(\mathrm{u}+\alpha, \mathrm{v}+\beta, w+\gamma)dudvdw$$

$$= \int_0^{\alpha}\int_0^{\beta}\int_0^{\gamma} e^{-(px+qy+rz)} F(x, y, z)dxdydz$$

$$+ e^{-(p\alpha+q\beta+r\gamma)}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-(pu+qv+rw)} F(\mathrm{u}, \mathrm{v}, w)dudvdw$$

$$= \int_0^{\alpha}\int_0^{\beta}\int_0^{\gamma} e^{-(px+qy+rz)} F(x, y, z)dxdydz + e^{-(p\alpha+q\beta+r\gamma)} f(p,q,r)$$

$$\therefore f(p,q,r) = \frac{1}{1-e^{-(p\alpha+q\beta+r\gamma)}}\int_0^{\alpha}\int_0^{\beta}\int_0^{\gamma} e^{-(px+qy+rz)} F(x, y, z)dxdydz.$$

Convolution and Convolution Theorem of the Triple Laplace Transforms

The convolution of $F(x, y, z)$ and $G(x, y, z)$ is denoted by $(F***G)(x,y,z)$ and defined by

$$(F***G)(x,y,z) = \int_0^x\int_0^y\int_0^z F(x-\alpha,\ y-\beta,\ z-\gamma)G(\alpha,\beta,\gamma)\,d\alpha\, d\beta\, d\gamma.$$

Theorem 7: Convolution Theorem: If $L_3\left[F(x,y,z)\right] = f(p,q,r)$ and $L_3\left[G(x,y,z)\right] = g(p,q,r)$, then

$$L_3\{(F***G)(x,y,z)\} = L_3\left[F(x,y,z)\right].L_3\left[G(x,y,z)\right] = f(p,q,r).g(p,q,r).$$

Or, equivalently,

$$L_3^{-1}\{f(p,q,r)g(p,q,r)\} = (F***G)(x,y,z).$$

Where $(F***G)(x,y,z)$ is defined by the triple integral which is often called the Convolution integral of $F(x,\ y,\ z)$ and $G(x,\ y,\ z)$. Physically, $(F***G)(x,y,z)$ represents the output of $F(x,\ y,\ z)$ and $G(x,\ y,\ z)$.

Proof: We have, by definition,

$$L_3\{(F***G)(x,y,z)\} = \int_0^\infty\int_0^\infty\int_0^\infty e^{-(px+qy+rz)}(F***G)(x,y,z)dxdydz$$

$$= \int_0^\infty\int_0^\infty\int_0^\infty e^{-(px+qy+rz)}\left[\int_0^x\int_0^y\int_0^z F(x-\alpha,\ y-\beta,\ z-\gamma)G(\alpha,\beta,\gamma)\,d\alpha\, d\beta\, d\gamma\right]dxdydz$$

which is, using the Heaviside unit step function,

$$\int_0^\infty\int_0^\infty\int_0^\infty e^{-(px+qy+rz)}\left[\int_0^\infty\int_0^\infty\int_0^\infty F(x-\alpha,\ y-\beta,\ z-\gamma)H(x-\alpha,\ y-\beta,\ z-\gamma)G(\alpha,\beta,\gamma)\,d\alpha\, d\beta\, d\gamma\right]dxdydz$$

$$= \int_0^\infty\int_0^\infty\int_0^\infty G(\alpha,\beta,\gamma)\,d\alpha\, d\beta\, d\gamma\left[\int_0^\infty\int_0^\infty\int_0^\infty e^{-(px+qy+rz)}F(x-\alpha,\ y-\beta,\ z-\gamma)H(x-\alpha,\ y-\beta,\ z-\gamma)\right]dxdydz$$

which is, by Theorem 5,

$$= \int_0^\infty\int_0^\infty\int_0^\infty e^{-(p\alpha+q\beta+r\gamma)} f(p,q,r)G(\alpha,\beta,\gamma)\,d\alpha\, d\beta\, d\gamma$$

$$= f(p,q,r)\int_0^\infty\int_0^\infty\int_0^\infty e^{-(p\alpha+q\beta+r\gamma)} G(\alpha,\beta,\gamma)\,d\alpha\, d\beta\, d\gamma$$

$$= f(p,q,r).g(p,q,r).$$

This completes the proof of the convolution theorem.

CONCLUSIONS

In this paper, Triple Laplace Transforms and Their properties, some simple example and applications are defined.

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